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Forced convection film condensation on a horizontal tube—effect of surface temperature variation

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Abstract—In free and forced convection condensation on a horizontal tube, measurements show that the surface temperature variation with angle approximately follows a cosine distribution. For forced convection condensation of steam, relatively low (with respect to isothermal-surface theoretical values) observed vapour-side heat-transfer coefficients have been attributed to surface temperature non-uniformity. A recent theoretical study of the effect of wall temperature variation in free convection condensation shows negligible effect on the average heat-transfer coefficient. In the present paper it is shown that, when a simple surface shear stress approximation, which gives satisfactory values for the mean heat-transfer coefficient for an isothermal tube, is adopted with variable wall temperature, *higher* mean surface heat-transfer coefficients are obtained. This seems to be in contradiction with results of conjugate (vapour-to-coolant) solutions which predict variable wall temperature and *lower* mean surface heat-transfer coefficients. Reasons for this apparent anomaly are advanced.

INTRODUCTION

FOR LAMINAR film condensation with uniform properties and negligible vapour velocity, the assumptions of the simple Nusselt theory [1] have been found in later and more complete studies to be generally valid. Nusselt's theory has also been well supported by experiment, particularly where care has been taken to avoid dropwise condensation and the effects of non-condensing gases. In view of the fact that the tube-wall temperature (taken to be uniform in the Nusselt theory and later refinements) has been observed to vary around the tube by amounts comparable with the mean temperature difference across the condensate film [2–9], the agreement between experiment and theory might seem somewhat surprising. Memory and Rose [7], however, have recently demonstrated that when a cosine distribution of wall temperature (i.e. close to measured wall temperature variation [7–9]) is used, together with the Nusselt approximations for the condensate film, the effect on the mean surface heat-transfer coefficient is negligible, despite the fact that the local film thickness and local heat flux depend strongly on the amplitude of the wall temperature variation. Thus, for free convection laminar film condensation on a horizontal tube the mean Nusselt number can be calculated with good accuracy from

$$\overline{Nu} = 0.728\left\{\frac{\rho^2 g h_{fg} d^3}{\mu k \Delta T}\right\}^{1/4}. \tag{1}$$

For forced convection condensation on a horizontal isothermal cylinder with vertical vapour downflow, Shekriladze and Gomelauri [10] obtained numerical solutions by using an approximate expression for the vapour shear stress on the condensate film together with the Nusselt assumptions. These were shown by Rose [11] to be represented to within 0.4% by:

$$\overline{Nu}\,\widetilde{Re}^{-1/2} = \frac{0.9+0.728F^{1/2}}{(1+3.44F^{1/2}+F)^{1/4}} \tag{2}$$

where $\overline{Nu}$ is the mean Nusselt number, $\widetilde{Re}$ is the 'two-phase Reynolds number' (vapour free-stream velocity and condensate properties) and F is a dimensionless parameter which measures the relative importance of gravity and vapour velocity for the motion of the condensate film.

A more recent solution using a better representation of the vapour shear stress [12, 13], including determination of, and allowance for, vapour boundary-layer separation, gives results that do not differ greatly from equation (2) except at low condensation rates (see also ref. [14]).

As shown in ref. [8], experimental data from several different investigations for condensation of steam in downflow over a horizontal tube indicate heat-transfer coefficients lower than predicted by equation (2) and in fair agreement with an empirical correlation of Fujii *et al.* [12],

$$\overline{Nu}\,\widetilde{Re}^{-1/2} = 0.96F^{1/5}. \tag{3}$$

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NOMENCLATURE

A	$a/\overline{\Delta T}$, see equations (4) and (9)
a	amplitude of surface temperature variation, see equation (4)
b	mean surface temperature, see equation (4)
d	diameter of tube
F	$(Gr/\tilde{Re}^2)\cdot(\mu h_{fg}/k\Delta T)$
Gr	$\rho^2 g d^3/\mu^2$
g	specific force of gravity
h_{fg}	specific enthalpy of evaporation
k	thermal conductivity of condensate
m	local condensation mass flux
$\overline{Nu}$	mean Nusselt number, see equation (17)
q	local heat flux
q^*	dimensionless local heat flux, see equation (15)
$\bar{q}$	mean heat flux, see equation (16)
R	radius of tube
$\tilde{Re}$	$U_\infty \rho d/\mu$
T_0	local surface temperature of tube
T_V	vapour temperature
U_∞	approach velocity of vapour
U'	tangential velocity at outer 'edge' of vapour boundary layer
u	tangential velocity in condensate film
y	radial distance from tube surface
z	dimensionless condensate film thickness, see equation (11)
z_0	value of z at $\phi = 0$
ΔT	local temperature difference across condensate film
$\overline{\Delta T}$	mean temperature difference across condensate film.

Greek symbols

δ	local condensate film thickness
μ	viscosity of condensate
ρ	density of condensate
ϕ	angle measured from top of tube.

Equations (1), (2) and (3) are compared in Fig. 1. (Note that equation (1) may be written $\overline{Nu}\,\tilde{Re}^{-1/2} = 0.728F^{1/4}$). It is seen that at high vapour velocity (low F) the experimental data represented by equation (3) fall well below theory (equation (2)).

As noted in ref. [9] the surface temperature variation is most pronounced when the vapour-side heat-transfer coefficient is much larger than that for the coolant-side. Thus, for the case of steam, where the thermal conductivity of the condensate is relatively high, the wall temperature depends more strongly on position around the tube. It has been suggested that this may explain the low experimental heat-transfer coefficients.

Honda and Fujii [15] have carried out conjugate (vapour-to-coolant) solutions using a constant inside (coolant) coefficient and solving the two-dimensional conduction problem in the tube wall. This approach led to lower mean vapour-side heat-transfer coefficients and approximately cosine surface temperature distributions, i.e. the theoretical results were in general accord with experiment. It is shown in the present paper that a cosine surface temperature distribution in forced convection condensation leads to *higher* average vapour-side heat-transfer coefficients when the Shekriladze–Gomelauri shear stress approximation is used.

FORCED CONVECTION CONDENSATION WITH VARIABLE WALL TEMPERATURE

Recent measurements [7, 16] for condensation of ethylene glycol in vertical downflow over a horizontal tube, confirm that surface temperature distributions

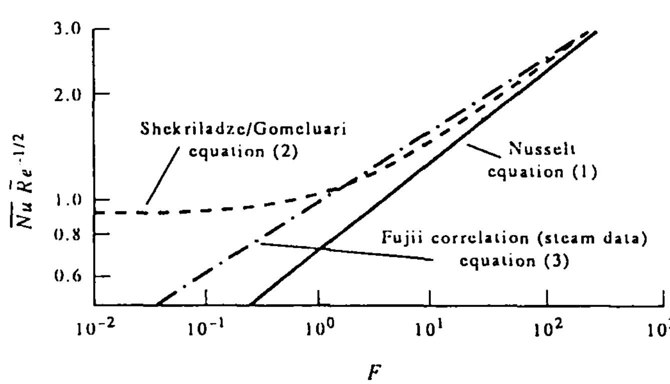

FIG. 1. Comparison of equations (1), (2) and (3).

can be very closely represented by equations of the form

$$T_0 = a\cos\phi + b. \tag{4}$$

For the purpose of investigating the effect of surface temperature variation, the approach used by Shekriladze and Gomelauri [10] may be adopted while using equation (4) rather than uniform wall temperature. Inertia, convection and pressure gradient in the condensate film are neglected. As in the Nusselt theory, the condensate film thickness is considered small in comparison with the tube radius. In the first instance gravity effects are also ignored, i.e. the motion of the condensate film is governed entirely by shear stress from the moving vapour and viscosity. A tangential momentum balance for the condensate film gives

$$\mu\frac{\partial^2 u}{\partial y^2} = 0. \tag{5}$$

Following the analysis of Shekriladze and Gomelauri [10], i.e. using the asymptotic (infinite condensation rate) approximation for the shear stress at the vapour-condensate interface and assuming potential flow outside the vapour boundary layer, the boundary condition

$$\frac{\partial u}{\partial y} = \frac{2mU_\infty \sin\phi}{\mu} \quad \text{at } y = \delta \tag{6}$$

is obtained†. m is the local condensation mass flux.

As in the Nusselt theory, integration of equation (5), with the conditions of zero velocity at the wall and equation (6), gives the velocity distribution across the condensate film

$$u = \frac{2mU_\infty \sin\phi}{\mu}\cdot y. \tag{7}$$

A mass balance for a condensate film element together with the assumption of radial conduction in the film gives

$$m = \frac{2\rho U_\infty k}{\mu d h_{fg}}\frac{\mathrm{d}}{\mathrm{d}\phi}(\delta\Delta T\sin\phi) \tag{8}$$

Where ΔT is the local temperature difference across the condensate film, assumed constant in the theory of Shekriladze and Gomelauri [10]. Here we use

$$\Delta T = \overline{\Delta T}(1 - A\cos\phi) \tag{9}$$

which follows from equation (4), where $\overline{\Delta T}$ is the mean temperature difference across the condensate film given by

$$\overline{\Delta T} = \frac{1}{\pi}\int_0^\pi \Delta T\,\mathrm{d}\phi \tag{10}$$

and A is a constant ($0 \leqslant A \leqslant 1$). Substituting for ΔT from equation (9) in equation (8) and replacing δ by the dimensionless film thickness

$$z = \frac{\rho U_\infty \delta^2}{\mu d} \tag{11}$$

gives

$$\frac{\mathrm{d}z}{\mathrm{d}\phi} + \frac{2z(\cos\phi - A\cos 2\phi)}{\sin\phi(1 - A\cos\phi)} - \frac{1}{\sin\phi} = 0. \tag{12}$$

Equation (12) may be solved for given values of A, subject to the boundary condition

$$\left(\frac{\mathrm{d}z}{\mathrm{d}\phi}\right)_{\phi=0} = 0. \tag{13}$$

Before proceeding to the results of numerical solutions, the dimensionless film thickness z_0 at $\phi = 0$ may be obtained directly from equations (12) and (13), i.e. $z_0 = 1/2$. It is interesting to note that the condensate film thickness at the forward stagnation point is the same regardless of the value of A and is not zero when $A = 1$, i.e. when $\Delta T_{\phi=0} = 0$.

Numerically-obtained solutions of equation (12) for various values of A, giving the dependence of the dimensionless condensate film thickness on angle, are shown in Fig. 2. It is seen that for uniform wall temperature (Shekriladze–Gomelauri case, $A = 0$), the film thickness increases continuously with ϕ. For larger values of A (stronger temperature variation around the tube) the film thickness at first decreases to a minimum before increasing. The fact that the film is relatively thick near $\phi = 0$ is due to the low surface shear stress at $\phi \to 0$ (see equation (6)). The shear stress is even smaller as $A \to 1$ since, in this case $m \to 0$ as $\phi \to 0$.

The local heat flux is given by

$$q = \frac{k\overline{\Delta T}(1 - A\cos\phi)}{\delta} \tag{14}$$

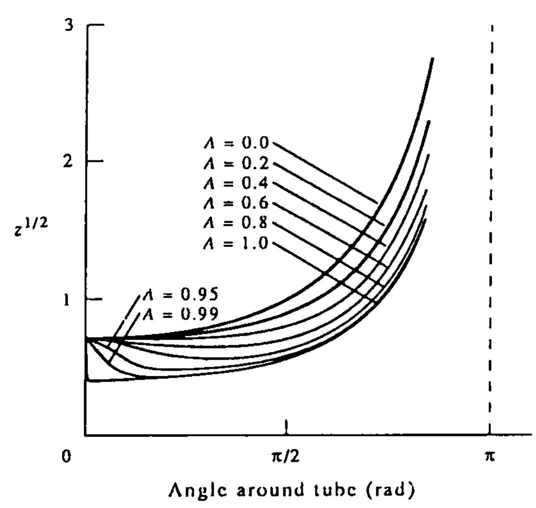

FIG. 2. Dependence of dimensionless condensate film thickness on angle for forced convection condensation.

† Equation (6) also requires $U' \gg u_{y=\delta}$.

which may be non-dimensionalised to give

$$q^* = q\left[\frac{\mu d}{\rho U_\infty k^2 \Delta T^2}\right]^{1/2} = (1 - A\cos\phi)z^{-1/2}. \quad (15)$$

The dependence of q^* on ϕ is shown in Fig. 3. It is seen that in the Shekriladze–Gomelauri case when $A = 0$, the heat flux falls continuously around the tube since the temperature difference is uniform and the condensate film thickens continuously. For large values of A, q^* is at first relatively small owing to the lower values of ΔT at smaller ϕ. As ϕ increases, q^* at first increases, owing to the increase in ΔT, until the effect of the thickening condensate film outweighs the effect of the increasing value of ΔT and q^* passes through a maximum before falling to zero when the film thickness becomes infinite as $\phi \to \pi$.

The mean heat flux for the tube is given by

$$\bar{q} = \frac{1}{\pi}\int_0^\pi q\, \mathrm{d}\phi \quad (16)$$

and the mean Nusselt number by

$$\overline{Nu} = \frac{\bar{q}}{\overline{\Delta T}}\frac{d}{k}. \quad (17)$$

Substituting from equations (15) and (16) we obtain

$$\overline{Nu}\,\widetilde{Re}^{-1/2} = \frac{1}{\pi}\int_0^\pi (1 - A\cos\phi)z^{-1/2}\,\mathrm{d}\phi. \quad (18)$$

Using numerical solutions of equation (12), the integral in equation (18) has been evaluated and the results, for various values of A, are given in Table 1. It may be seen from Table 1 that $\overline{Nu}\,\widetilde{Re}^{-1/2}$ *increases* from the Shekriladze–Gomelauri value of 0.9, with

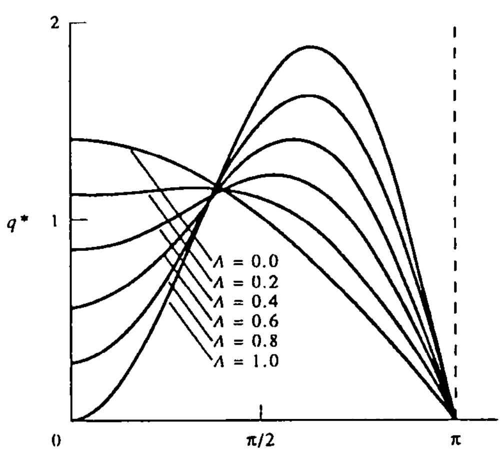

FIG. 3. Dependence of dimensionless heat flux on angle for forced convection condensation.

Table 1. Dependence of mean Nusselt number on a forced convection film condensation

A	$\overline{Nu}\,\widetilde{Re}^{-1/2}$
0	0.900
0.2	0.906
0.4	0.924
0.6	0.953
0.8	0.992
1.0	1.040

increasing A, to a maximum value of 1.04 at $A = 1$. Thus for a cosine temperature distribution, the mean Nusselt number is independent of amplitude for free convection (see Memory and Rose [7]) and *increases* in forced convection condensation when the Shekriladze–Gomelauri surface shear stress approximation is used. We thus anticipate intermediate behaviour for the case of combined free and forced convection.

COMBINED FORCED AND FREE CONVECTION

For vertical vapour downflow with gravity included, a tangential momentum balance for the condensate film gives

$$\mu\frac{\partial^2 u}{\partial y'^2} + \rho g \sin\phi = 0. \quad (19)$$

With the boundary condition given by equation (6) and proceeding as before we obtain, for the local condensation mass flux

$$m = \frac{2\rho^2 g}{3\mu d}\frac{\mathrm{d}}{\mathrm{d}\phi}(\delta^3 \sin\phi) + \frac{2\rho U_\infty k}{\mu d h_{fg}}\frac{\mathrm{d}}{\mathrm{d}\phi}(\delta \Delta T \sin\phi). \quad (20)$$

Substituting for ΔT from equation (9) and non-dimensionalising, equation (20) becomes

$$(Fz + 1 - A\cos\phi)\sin\phi\frac{\mathrm{d}z}{\mathrm{d}\phi} + 2z\left(\frac{Fz}{3}\cos\phi + \cos\phi - A\cos 2\phi\right) - 1 + A\cos\phi = 0. \quad (21)$$

Again, the symmetry condition (equation (13)) enables us to determine the film thickness at the top of the tube without solving the differential equation. In this case

$$z_0 = \frac{-1 + \sqrt{(1 + 2F/\{3(1-A)\})}}{2F/\{3(1-A)\}}. \quad (22)$$

It may be seen that for $F \to \infty$ (free convection limit) $z_0 = 0$ when $A = 1$ in agreement with the result found

Table 2. Dependence of mean Nusselt number on A and F for combined free and forced convection condensation

	$\overline{Nu}/\overline{Nu}_{\text{eqn}(2)}$					
A	$F = 100$	$F = 10$	$F = 1$	$F = 0.1$	$F = 0.01$	$F = 0.001$
0	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.01	1.01	1.01
0.4	1.00	1.00	1.00	1.02	1.03	1.03
0.6	1.00	1.01	1.02	1.05	1.06	1.06
0.8	1.01	1.02	1.05	1.09	1.10	1.10
1.0	1.01	1.04	1.09	1.14	1.16	1.16

by Memory and Rose [7], while for $F \to 0$ (shear controlled limit), $z_0 = 1/2$ as found above.

Equation (21) may be solved numerically, with boundary condition equation (13), to give the dependence of z on ϕ for given values of A and F. The local heat flux and local dimensionless heat flux are again given by equations (14) and (15). The mean Nusselt number may be obtained using equation (18) with the values of z from the solution of equation (21).

A selection of the results obtained is given in Table 2 and Fig. 4. Table 2 gives values of Nusselt number normalized by those given by equation (2). As noted earlier, equation (2) is a very close approximation to the result for the isothermal wall case. It is seen that when $A = 0$, the normalised Nusselt number is unity for all F as expected. For higher values of A the normalised Nusselt number remains close to unity at high F where gravity dominates, in accordance with the results of Memory and Rose [7]. At low F the normalised Nusselt number increases with increasing A in accordance with the present results for the shear controlled limit.

The same conclusions can be drawn from Fig. 4. At high values of F (gravity controlled case) the curves converge to the Nusselt solution for all values of A. At low values of F (shear controlled case), $\overline{Nu}\,\widetilde{Re}^{-1/2}$ increases with increasing values of A.

CONCLUSION

Comparison of the present results, obtained using the Shekriladze and Gomelauri [10] surface shear stress approximation, with the conjugate solutions for steam obtained by Honda and Fujii [15] where a more realistic shear stress model was used, indicates that the Shekriladze and Gomelauri approach, although giving satisfactory values of the mean heat-transfer coefficient for an isothermal tube, leads to overestimates when the surface temperature of the tube varies significantly.

The most probable explanation would seem to be that equation (6) underestimates the surface shear stress on the upper part of the tube where ΔT, and hence q, are relatively small, and overestimates the surface shear stress on the lower part of the tube (after vapour boundary layer separation) where ΔT is relatively high. The net effect of using equation (6) is therefore to increase the calculated *total* heat transfer and hence the mean heat-transfer coefficient.

A second, though probably less important, factor contributing to the discrepancy between the present results and the conjugate solutions for steam of Honda and Fujii [15], may be error in evaluating $\overline{\Delta T}$ from the temperature distributions calculated in the conjugate theory. The mean vapour-to-surface temperature difference is given by

$$\overline{\Delta T} = \frac{1}{\pi k}\int_0^{\pi} q\delta \,\mathrm{d}\phi. \tag{23}$$

As $\phi \to \pi$ the calculated values of q and δ become increasingly unreliable since the assumption $\delta \ll R$ becomes invalid. That this leads to error in uniform

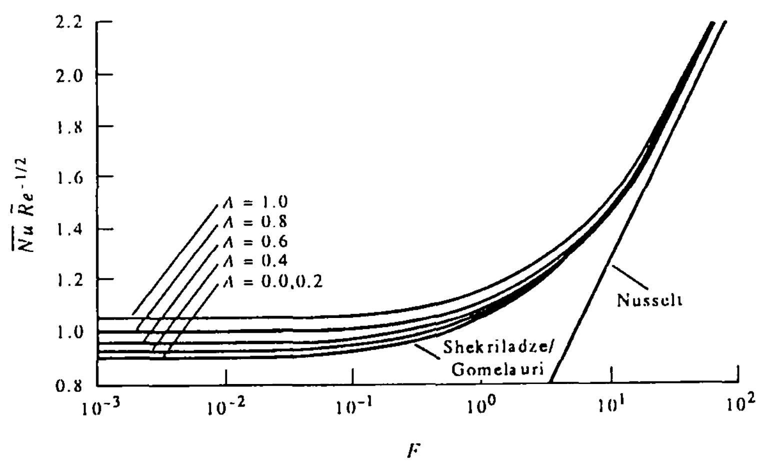

FIG. 4. Dependence of $\overline{Nu}\,\widetilde{Re}^{-1/2}$ on parameter A for combined forced and free convection.

q solutions, where the integral in equation (23) is significantly affected by large and inaccurate values of δ near $\phi = \pi$, was discussed in ref. [14]. In the general case the extent to which the erroneous values of the $q\delta$ product near $\phi = \pi$ affect the integral in equation (23) is not evident, since while $\delta \rightarrow \infty$, $q \rightarrow 0$.

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